~~FIFTH~~ SIXTH ANNUAL MICHIGAN MATHEMATICS PRIZE COMPETITION

Sponsored by

The Michigan Section of The Mathematical Association of America,
Michigan Colleges, Universities, Professional Organizations, and Industries

DECEMBER 13, 1962

PART II

INSTRUCTIONS
(To Be Read Aloud to Class by Supervisor or Proctor)

1. Part II is not a multiple choice test, but consists of problems and proofs. You will be allowed 60 minutes for five questions.

2. As in Part I, you are not expected to finish the five questions, so attempt to solve first those which interest you most.

3. This is your opportunity to make a good impression, and the Examiners will take into account the way in which you attack a question and the way in which you explain your solution.

4. Make your preliminary calculations on blank paper supplied by your school, and write your solution in the space under the question in the examination booklet. If more space is needed, ask your supervisor for extra paper.

5. As in Part I, stop when your supervisor announces that the sixty minutes are up. As before, your supervisor is not permitted to violate the rules by answering any questions.

NAME ______________________ HIGH SCHOOL GRADE __________
(please print)

HIGH SCHOOL ______________________ CITY ______________

HIGH SCHOOL NUMBER __________

COLLEGES AND UNIVERSITIES TO WHICH ADMISSION APPLICATIONS ARE BEING SUBMITTED ______________________________

SCHOLARSHIPS APPLIED FOR ______________________________

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PART II

1. Consider this statement: An equilateral polygon circumscribed about a circle is also equiangular.

   Decide whether this statement is a true or false proposition in euclidean geometry. If it is true, prove it; if false, produce a counterexample.

PART II

2. Show that the fraction $\frac{x^2 - 3x + 1}{x - 3}$ has no value between 1 and 5, for any real value of x.

PART II

3. A man walked a total of 5 hours, first along a level road and then up a hill, after which he turned around and walked back to his starting point along the same route. He walks 4 miles per hour on the level, three miles per hour uphill, and r miles per hour downhill. For what values of r will this information uniquely determine his total walking distance?

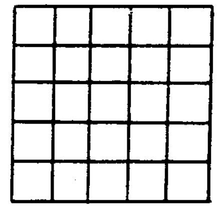
PART II

4. A point P is so located in the interior of a rectangle that the distance of P from one corner is 5 yards, from the opposite corner is 14 yards, and from a third corner is 10 yards. What is the distance from P to the fourth corner?

5. Each small square in the 5 by 5 checkerboard shown has in it an integer according to the following rules:

   i. Each row consists of the integers 1, 2, 3, 4, 5 in some order.

   ii. The order of the integers down the first column is the same as the order of the integers, from left to right, across the first row and similarly for any other column and the corresponding row.



Prove that the diagonal squares running from the upper left to the lower right contain the numbers 1, 2, 3, 4, 5 in some order.