

School No.

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Student No.

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FOURTEENTH ANNUAL
MICHIGAN MATHEMATIC PRIZE COMPETITION

sponsored by

The Michigan Section of the Mathematical Association of America,
Michigan Colleges and Universities, Professional Organizations, and
Industries.

PART II

December 9, 1970

INSTRUCTIONS

(To be read aloud to class by supervisor or proctor)

1. Record, in the upper lefthand corner of this page, your school number and your student number. This is the only way to identify this test booklet with your name. Please do not write your name on the booklet.
2. Part II consists of problems and proofs. You will be allowed 100 minutes for the five questions.
3. You are not expected to solve all questions completely. Look over all problems and work first on those which interest you the most.
4. Each problem is on a different page. You should show most of your work on that page. If it is necessary to use additional paper for your answer, please indicate clearly your identification number and problem number in the upper lefthand corner of each sheet.
5. If you are unable to solve a particular problem, partial credit might be given for indicating a possible procedure or an example to illustrate the ideas involved.
6. You are advised to consider specializing or generalizing any problem where it seems appropriate. Sometimes an examination of special cases may generate ideas of how to attack the problem. On the other hand, a careful stated generalization may justify additional credit provided you give an explanation of why the generalization might be true.
7. Your supervisor is not permitted to violate the rules by answering any questions. When the supervisor announces that the 100 minutes are up, please cease work immediately and insert all significant extra paper, including the questionnaire form, into the booklet. It is not necessary to return scratch paper on which routine numerical calculations were made.

Score
 1 2 3 4 5 Total

1. Show that the $n \times n$ determinant

$$\begin{vmatrix} 1+x & 1 & 1 & \cdot & \cdot & \cdot & 1 \\ 1 & 1+x & 1 & \cdot & \cdot & \cdot & 1 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 1 & 1 & \cdot & \cdot & \cdot & \cdot & 1+x \end{vmatrix}$$

has the value zero when $x = -n$.

2. Let $c > a \geq b$ be the lengths of the sides of an obtuse triangle. Prove that $c^n = a^n + b^n$ for no positive integer n .

3. Suppose that $p_1 = p_2^2 + p_3^2 + p_4^2$, where p_1, p_2, p_3 , and p_4 are primes. Prove that at least one of p_2, p_3, p_4 is equal to 3.

4. Suppose X and Y are points on the boundary of the triangular region ABC such that the segment XY divides the region into two parts of equal area. If XY is the shortest such segment and $AB = 5$, $BC = 4$, $AC = 3$ calculate the length of XY .

Hint: Of all triangles having the same area and same vertex angle the one with the shortest base is isosceles.

Clearly justify all claims.

5. Find all solutions of the following system of simultaneous equations:

$$x + y + z = 7, \quad x^2 + y^2 + z^2 = 31, \quad x^3 + y^3 + z^3 = 154.$$

The following Michigan companies and professional organizations have made contributions to the scholarship fund for this competition.

Aeroquip Corporation, Jackson
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The Michigan Mathematics Prize Competition is an activity of the Michigan Section of the Mathematical Association of America.

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