

## TWENTY-SEVENTH ANNUAL

## MICHIGAN MATHEMATICS PRIZE COMPETITION

sponsored by the

MATHEMATICAL ASSOCIATION OF AMERICA, MICHIGAN SECTION

PART II

December 7, 1983

## INSTRUCTIONS

(to be read aloud to class by supervisor or proctor)

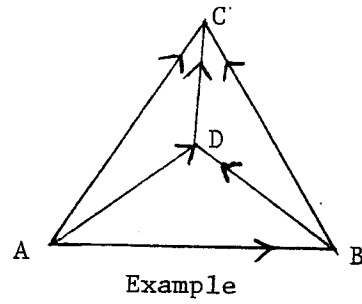
1. Carefully record your six digit MMPC code number in the upper righthand corner of this page. This is the only way to identify you with this test booklet. PLEASE DO NOT WRITE YOUR NAME ON THIS BOOKLET.
2. Part II consists of problems and proofs. You will be allowed 100 minutes for the five questions. To receive full credit for a problem, you are expected to justify your answer.
3. You are not expected to solve all problems completely. Look over all the problems and work first on those which interest you the most.
4. Each problem is on a different page. You should show most of your work on that page. If it is necessary to use additional paper for your answer, indicate this on the exam page and write your identification number and the problem in the upper righthand corner of each additional sheet.
5. If you are unable to completely solve a particular problem, partial credit might be given for indicating a possible procedure or an example to illustrate the ideas involved. If you have difficulty understanding what is required in a given problem, note this on your answer sheet and attempt to make a nontrivial restatement of the problem. Then try to solve the restated problem.
6. You are advised to consider specializing or generalizing any problem where it seems appropriate. Sometimes an examination of special cases may generate ideas of how to attack the problem. On the other hand, a carefully stated generalization may justify additional credit provided you give an explanation of why the generalization might be true.
7. The competition rules do not allow anyone to answer any questions. The use of notes, reference material, computational aids, or any other aid is prohibited. When the supervisor announces that the 100 minutes are up, please cease work immediately and insert all significant extra paper into the booklet. It is not necessary to return scratch paper on which routine numerical calculations were made.
8. You may now open the test booklet and begin.

Score	_____	_____	_____	_____	_____	_____
	1	2	3	4	5	TOTAL

1. Find the largest integer which is a factor of all numbers of the form  $n(n + 1)(n + 2)$  where  $n$  is any positive integer with unit digit 4. Prove your claims.

2. Each pair of the towns A, B, C, D is joined by a single one way road. See example. Show that for any such arrangement a salesman can plan a route starting at an appropriate town that enables him to call on a customer in each of the towns.

Note that it is not required that he return to his starting point.



3. A and B are two points on a circular race track. One runner starts at A running counter clockwise, and, at the same time, a second runner starts from B running clockwise.

They meet first 100 yds from A, measured along the track. They meet a second time at B and the third time at A. Assuming constant speeds, how long is the track?

4. A and B are points on the positive x and positive y axis, respectively, and C is the point (3,4). Prove that the perimeter of  $\triangle ABC$  is greater than 10.

Suggestion: Reflect!!

5. Let  $A_1, A_2, \dots, A_8$  be a permutation of the integers  $1, 2, \dots, 8$  so chosen that the eight sums  $9 + A_1, 10 + A_2, \dots, 16 + A_8$  and the eight differences  $9 - A_1, 10 - A_2, \dots, 16 - A_8$  together comprise 16 different numbers.

Show that the same property holds for the eight numbers in reverse order. That is, show that the 16 numbers  $9 + A_8, 10 + A_7, \dots, 16 + A_1$  and  $9 - A_8, 10 - A_7, \dots, 16 - A_1$  are also pairwise different.

The Michigan Mathematics Prize Competition is an activity of the Michigan  
Section of the Mathematical Association of America

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ACKNOWLEDGEMENTS

The following industries and professional organizations have provided generous  
financial support to this competition.

Burroughs Corporation  
Michigan Bell Telephone  
Michigan Council of Teachers of Mathematics  
Kuhlman Corporation

The Michigan Association of Secondary School Principals has placed this compe-  
tition on the Approved List of Michigan Contests and Activities.