

**THIRTY-THIRD ANNUAL  
MICHIGAN MATHEMATICS PRIZE COMPETITION**

sponsored by  
The Michigan Section of the Mathematical Association of America

**Part I**

October 11, 1989

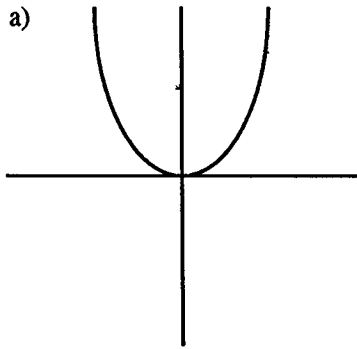
**INSTRUCTIONS**

(to be read aloud to the students by the supervisor or proctor)

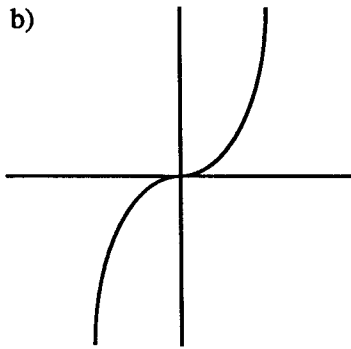
1. Your answer sheet will be graded by machine. Please read and follow carefully the instructions printed on the answer sheet. **Check to insure that your six-digit code number has been recorded correctly.** Do not make calculations on the answer sheet. Fill in circles completely and darkly.
2. Do as many problems as you can in the 100 minutes allowed. When the proctor requests you to stop, please quit working immediately and turn in your answer sheet.
3. Essentially all of the problems require some figuring. Do not be hasty in your judgements. For each problem you should work out ideas on scratch paper before selecting the answer.
4. You may be unfamiliar with some of the topics covered in this examination. You may skip over these and return to them later if you have time. Your score on the test will be the number correct. You are advised to guess an answer in those cases where you cannot determine an answer.
5. In each of the questions, five different possible responses are provided. In some cases the fifth alternative is listed "e) none of these" or "e) none of the above". If you believe none of the first four alternatives to be correct, mark e) in such cases.
6. No one is permitted to explain to you the meaning of any question. Do not request anyone to break the rules of the competition. The use of books, tables, slide rules, electronic calculators, notes or any other aid is prohibited. If you have questions concerning the instructions, ask them now.
7. You may now open the test booklet and begin.

1. The graph of  $f(x) = x|x|$  looks most nearly like

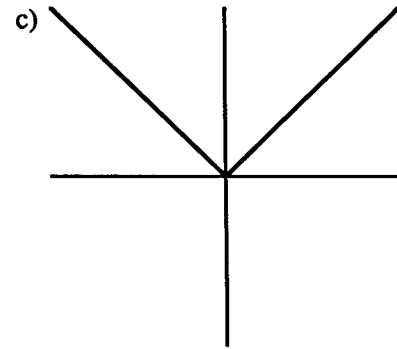
a)



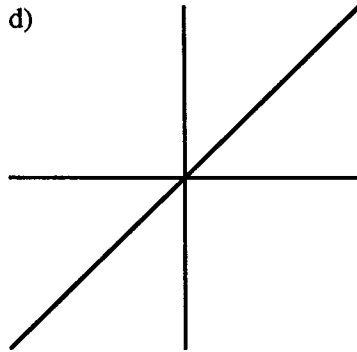
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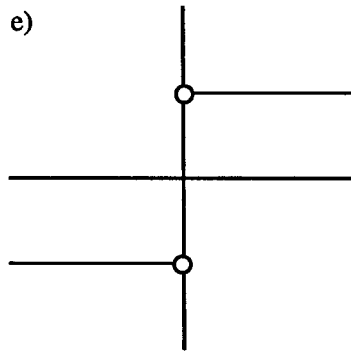
c)



d)



e)



2. The smallest positive angle  $\theta$  satisfying the equation  $\cos^2 \theta - \sin^2 2\theta = 0$  is

a) 0 degrees

b) 90 degrees

c) 180 degrees

d) 270 degrees

e) None of these is the smallest solution

3. A circle is circumscribed about an isosceles right triangle whose short side has length 3. What is the circumference of the circle?

a)  $\frac{9\pi}{2}$ b)  $6\pi$ c)  $3\sqrt{2}\pi$ d)  $\frac{3\sqrt{2}\pi}{2}$ 

e) None of these

4. A man has four pairs of socks from which he selects two socks at random. What is the probability that the two socks will match?

a)  $\frac{1}{4}$ b)  $\frac{1}{6}$ c)  $\frac{1}{7}$ d)  $\frac{1}{8}$ 

e) None of these

5. The value of  $4 + 8 + 12 + 16 + 20 + \dots + 100$  is

a) 1280

b) 1300

c) 1320

d) 1340

e) None of these

6. The number  $(x - 4)$  is how much less than the number  $(x + 4)$ ?

a) 4

b) 8

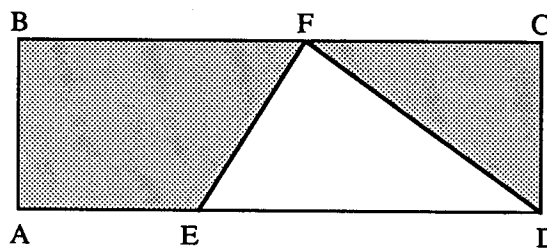
c)  $x - 4$ d)  $x - 8$ e)  $2x$

7. The expression  $\frac{1}{3\sqrt{2} - 2\sqrt{3}}$  is also equal to
- a)  $\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}}$     b)  $\frac{3\sqrt{2} - 2\sqrt{3}}{6}$     c)  $\frac{1}{3\sqrt{2}} - \frac{1}{2\sqrt{3}}$     d)  $\frac{\sqrt{3} - \sqrt{2}}{\sqrt{6}}$     e) None of these
8. If  $r_1$ ,  $r_2$ , and  $r_3$  are the roots of the equation  $x^3 - 5x + 1 = 0$ , then  $r_1 + r_2 + r_3$  equals
- a) -5    b) 0    c) 1    d) 5    e) None of these
9. The value of  $\cos(\arccos(-2))$  is
- a) -2    b) 0    c) 2    d)  $-\frac{1}{2}$     e) Undefined
10. If the numbers  $a$ ,  $\frac{1}{a}$ ,  $b$ ,  $\frac{1}{b}$ ,  $c$ , and  $\frac{1}{c}$  are all integers, which could **not** be a value of  $a + b + c$ ?
- a) -3    b) -1    c) 1    d) 2    e) 3
11. A circle has diameter 30, and an arc AB of length  $\frac{15\pi}{2}$ . The length of the chord AB is
- a)  $\frac{15\pi}{2}$     b)  $\frac{15}{\sqrt{2}}$     c)  $30\sqrt{2}$     d)  $\frac{15\pi}{2\sqrt{2}}$     e) None of these
12. For what value of  $x$  is  $4^{3x} = 16^{2x+1}$ ?
- a) -2    b) 0    c) 1    d) 2    e) None of these
13. Three distinct real numbers sum to 89. The difference between the larger two is twice the difference between the smaller two. Yet the smaller two sum to 3 more than the largest number. What is the smallest number?
- a) 9.25    b) 13    c) 16.6    d) 19    e) None of these
14. The expression  $\log_{10} 125 + \log_{10} 8$  is equal to
- a) 2    b) 3    c) 4    d) 5    e) None of these
15. For all  $x > 0$  except  $x = 1$ , and for all  $y > 0$  except  $y = 1$ , the value of  $(\log_y x)(\log_x y)$  is
- a) 1    b)  $\frac{1}{4}$     c)  $\frac{1}{xy}$     d)  $\log_{x+y} xy$     e) None of these
16. Define  $\langle m \rangle = \frac{m(m+1)}{2}$  for all integers  $m$ , and let  $n = \langle 3 \rangle$ . Then  $\langle n \rangle$  equals
- a) 6    b) 12    c) 21    d) 42    e) None of these
17. The polynomial equation  $x^6 - 2x^4 - x^2 + 2 = 0$  has
- a) No real roots    b) Exactly two real roots  
c) Exactly four real roots    d) Exactly six real roots    e) None of these



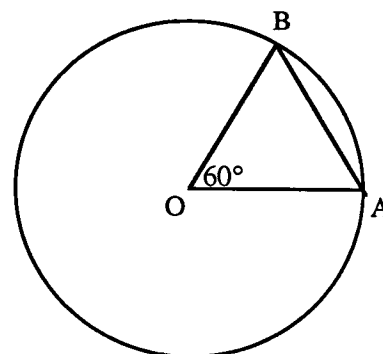
25. Mack and Sean can complete a job in 2 hours when working together. If Mack requires 3 hours to do the job alone, how many hours does Sean need to do the job alone?  
 a) 2                      b) 3                      c) 4                      d) 5                      e) 6

26. In the rectangle ABCD, the ratio of ED to AD is 2 to 3. The area of triangle  $\triangle EFD$  is 24, and F is an arbitrary point on the side BC. The area of the shaded portion is  
 a) 8                      b) 36                      c) 48  
 d) 72                      e) None of these



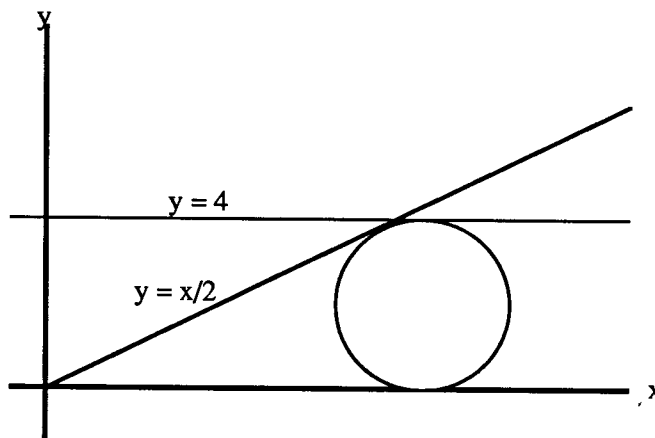
27. The graphs of  $|y| = x + 1$  and  $(x - 1)^2 + y^2 = 4$  intersect in how many points?  
 a) 1                      b) 2                      c) 3                      d) 4                      e) None of these

28. In the figure, O is the center of the circle. If  $AB = 5$ , what is the area of the circle?  
 a)  $5\pi$   
 b)  $10\pi$   
 c)  $20\pi$   
 d)  $25\pi$   
 e) The solution cannot be found from the information given.



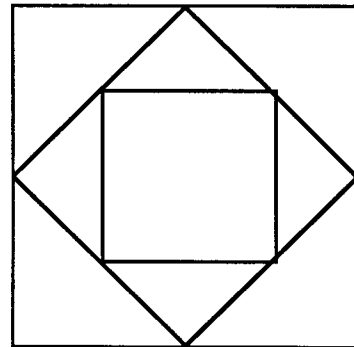
29. There is a real number  $x$  whose value is given by  $x = \sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{1 + \dots}}}}$ . Then  $x$  equals  
 a) 1                      b)  $\sqrt{2}$                       c)  $\sqrt{5}$                       d)  $\frac{1 + \sqrt{5}}{2}$                       e) None of these

30. The  $x$ -coordinate of the center of the circle in the first quadrant (see figure) tangent to the lines  $y = \frac{1}{2}x$ ,  $y = 4$ , and the  $x$ -axis is  
 a)  $4 + 2\sqrt{5}$   
 b)  $4 + \frac{8\sqrt{5}}{5}$   
 c)  $8 + 2\sqrt{5}$   
 d)  $2 + \frac{6\sqrt{5}}{5}$   
 e) None of these



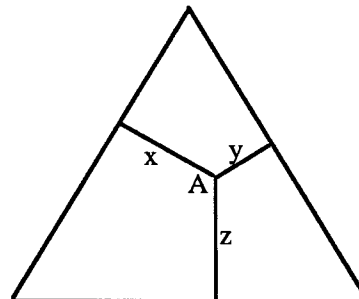
31. If the radius of a circle is decreased by 40 %, by what percentage is the area decreased?  
 a) 16 %                      b) 36 %                      c) 40 %                      d) 64 %                      e) None of these

32. Given three concentric squares in which the corners of each inner square bisect the sides of the next larger square, what is the ratio of the area of the largest square to the area of the smallest square?  
 a) 3 to 1  
 b) 4 to 1  
 c) 8 to  $\sqrt{2}$   
 d) The ratio will vary depending on the size of the innermost square.  
 e) None of these

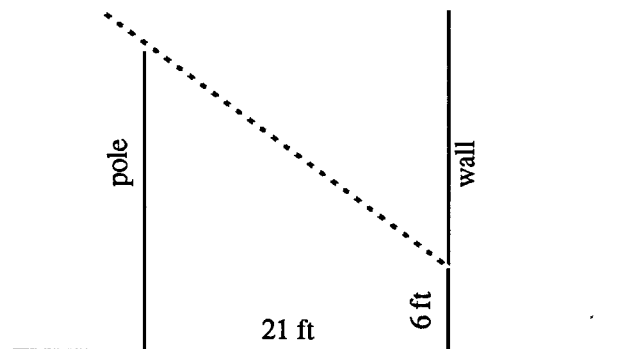


33. If we approximate  $\frac{1}{1 - \cos \theta}$  by  $1 + \cos \theta + \cos^2 \theta$ ,  
 then the ratio of the error made to the correct value is  
 a)  $\cos^3 \theta$                       b)  $\cos^2 \theta$                       c)  $\frac{\cos^3 \theta}{1 - \cos \theta}$                       d)  $\frac{\cos^2 \theta}{1 - \cos \theta}$                       e) None of these

34. A certain point A inside an equilateral triangle with perimeter P is at distances x, y, and z from the three sides, respectively. The sum  $x + y + z$  is always  
 a) P                      b)  $\frac{P}{\sqrt{3}}$   
 c)  $\frac{2P}{\sqrt{3}}$                       d)  $\frac{P}{2\sqrt{3}}$   
 e) The sum cannot be found from the information given.



35. A flagpole casts a shadow 6 feet high on a wall 21 feet away from its base. How high is the flagpole?  
 a) 7 feet                      b) 21 feet  
 c) 27 feet                      d)  $6 + 7\sqrt{3}$  feet  
 e) Cannot be determined from the information given.



36. How many angles between 0 degrees and 360 degrees satisfy  $\cos x = \sin 2x$ ?

- a) 0                      b) 1                      c) 2                      d) 3                      e) 4

37. Which of the numbers listed below is not a divisor of  $2^{30} - 1$ ?

- a)  $2^5 - 1$               b)  $2^5 + 1$               c)  $2^6 - 1$               d)  $2^{10} - 1$               e)  $2^{10} + 1$

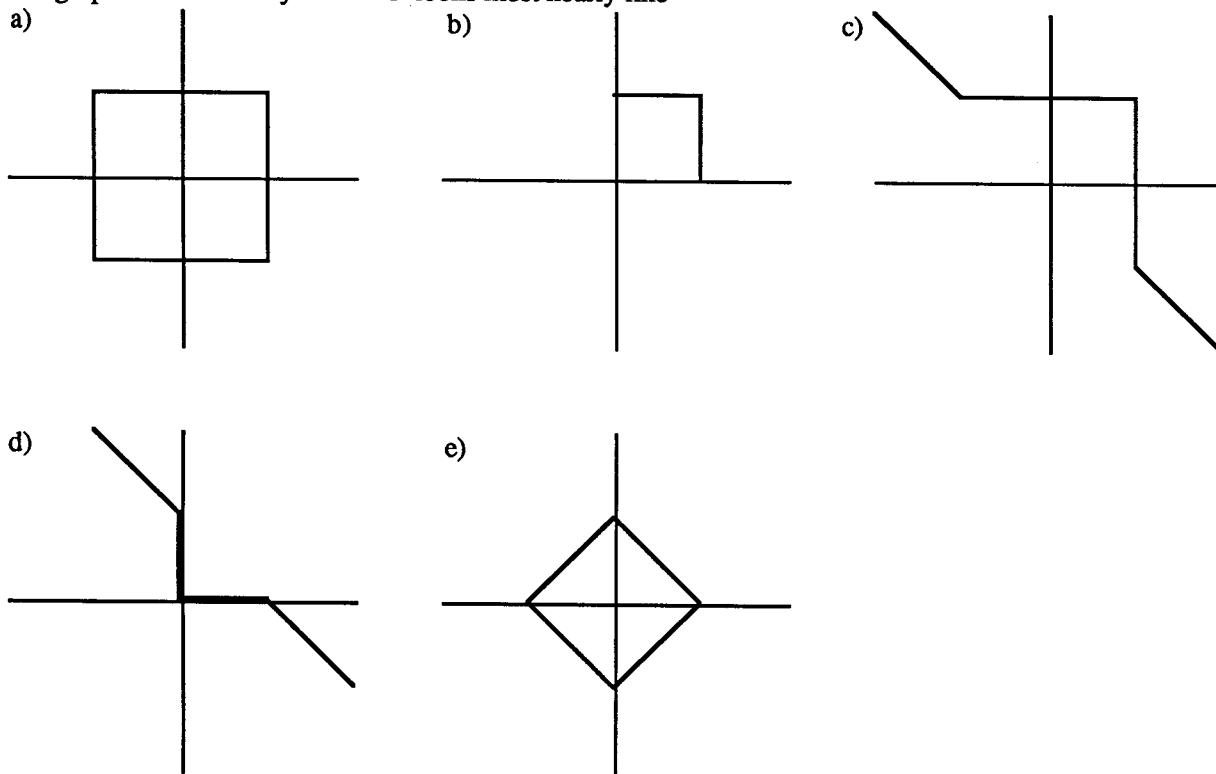
38. Define  $f(x) = 1 - \frac{1}{x}$  and let  $y = f(f(f(x)))$ . For all real  $x \neq 0$  or 1, we can conclude that

- a)  $y = x$               b)  $y > x$               c)  $y < x$               d)  $y = (1 - \frac{1}{x})^3$   
 e) None of these is true for all real  $x \neq 0$  or 1.

39. What is the coefficient of  $x^4$  in the expansion of  $(3x^2 - 2)^5$ ?

- a) -810                  b) -720                  c) -72                  d) 720                  e) None of these

40. The graph of  $x^{1989} + y^{1989} = 1$  looks most nearly like



The Michigan Mathematics Prize Competition is an activity of the Michigan Section of the  
Mathematical Association of America

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