

**FORTIETH ANNUAL
MICHIGAN MATHEMATICS PRIZE COMPETITION**

Sponsored by
The Michigan Section of the Mathematical Association of America

Part I

October 16, 1996

INSTRUCTIONS

(to be read aloud to the students by the supervisor or proctor)

1. Your answer sheet will be graded by machine. Please read and follow carefully the instructions printed on the answer sheet. **Check to insure that your six-digit code number has been recorded correctly.** Do not make calculations on the answer sheet. Fill in circles completely and darkly.
2. Do as many problems as you can in the 100 minutes allowed. When the proctor requests you to stop, please quit working immediately and turn in your answer sheet.
3. Essentially all of the problems require some figuring. Do not be hasty in your judgments. For each problem you should work out ideas on scratch paper before selecting the answer.
4. You may be unfamiliar with some of the topics covered in this examination. You may skip over these and return to them later if you have time. Your score on the test will be the number correct. You are advised to guess an answer in those cases where you cannot determine an answer.
5. In each of the questions, five different possible responses are provided. In some cases the fifth alternative is listed "**e) none of these**". If you believe none of the first four alternatives to be correct, mark **e)** in such cases.
6. No one is permitted to explain to you the meaning of any question. Do not request anyone to break the rules of the competition. Scientific and graphing calculators are allowed on Part I this year, but the use of books, tables, slide rules, notes or any other aid is prohibited. If you have questions concerning the instructions, ask them now.
7. You may now open the test booklet and begin.

1. Suppose $f(x) = \sqrt{x-1}$ and $g(x) = x^2 + 1$. Find and simplify $f(g(x-1))$.

- (a) $|x-1|$ (b) $x-1$ (c) $\sqrt{x^2-1}$ (d) $\sqrt{x-2}$ (e) None of these

2. $\frac{ax+b}{cx+b} =$

- (a) $\frac{a+b}{c+b}$ (b) $\frac{ax+1}{cx+1}$ (c) $\frac{a}{c}$ (d) $\frac{a}{c} + 1$ (e) none of these

3. A pig is worth a sheep and a dog. Two pigs are worth three dogs. How many sheep is a pig worth?

- (a) 2 (b) 3 (c) $\frac{1}{3}$ (d) $1\frac{1}{2}$ (e) 1 sheep and 1 dog.

4. What are the last two digits of 1996^{1996} ?

- (a) 16 (b) 36 (c) 56 (d) 76 (e) 96.

5. You are playing with a spirograph set and pick out a big ring with 96 teeth on the inside and an inner wheel with 21 teeth on it. How many times around the inside of the big ring do you go until you start to repeat the pattern?

- (a) 1 (b) 7 (c) 14 (d) 15 (e) 21.

6. You roll 1728 pairs of dice. For how many of those 1728 pairs do you expect the sum to be exactly 7?

- (a) 344 (b) 288 (c) 212 (d) 157 (e) 144

7. When you bicycle to your friend's house, your average velocity is 10 mph. You return by the same route, but your average velocity is only 8 mph on the return trip. What is your average velocity for the round trip (to the nearest .01 mph)?

- (a) the problem cannot be done without knowing the distance
(b) 9.00 mph (c) 8.92 mph (d) 8.89 mph (e) 8.84 mph.

8. How many sets are there of four consecutive integers a, b, c, d (where $a < b < c < d$), for which $a^2 + d^2 = b^2 + c^2$?

- (a) 0 (b) 1 (c) 2 (d) 4 (e) infinitely many.

9. 100 adult rabbits (50% male and 50% female) are placed on a fertile island with plenty of vegetation and no predators. The average adult female rabbit has a litter every 3 weeks, young rabbits are adults after 3 weeks, hence young females have their first litter at the age of 6 weeks, the average litter size is 6 babies, each baby is equally likely to be male or female, and no rabbits die during the period. Estimate the rabbit population 12 weeks later.
- (a) 1,000 (b) 1,200 (c) 4,000 (d) 13,300 (e) 25,600.
10. Suppose that $a = \sin t^\circ$. Express $\cos (90 - t)^\circ$ in terms of a .
- (a) $\pm\sqrt{1-a^2}$ (b) $\sqrt{1-a^2}$ (c) $1-a$ (d) $-a$ (e) a
11. How many distinguishable ways are there to seat six people, including one pair of identical twins at a round table? [We assume that rotating the seats any number in either direction leads to indistinguishable seatings. In addition, if the identical twins were to change seats with each other an indistinguishable seating would result.]
- (a) 360 (b) 240 (c) 120 (d) 60 (e) 11
12. You replace your truck tires with larger ones. The original tires had radius 0.3 meters and the replacement tires had radius 0.5 meters. The speedometer, which you do not modify, measures the speed by counting the number of rotations that the axle makes per minute. What is the maximum speedometer reading that you can allow if you want to stay within the speed limit of 65 miles per hour?
- (a) 39 mph (b) 43 mph (c) 53 mph (d) 65 mph (e) 108.33 mph
13. If $x*y = x^y$ and $x\#y = x+2y$, find $[(3*2)\#5]\#1$.
- (a) 21 (b) 20 (c) 19 (d) 18 (e) None of these
14. The grading system at a school is as follows:
A = 4, B = 3, C = 2, D = 1, F = 0. A plus adds 0.3 and a minus subtracts 0.3 (so that B- = 2.7 and B+ = 3.3). A particular student has 5 credits of A-, 3 credits of C+ and 4 credits of B-. What is this student's grade-point average to the nearest .01?
- (a) 36.20 (b) 3.17 (c) 3.02 (d) 3.00 (e) 2.90
15. $\log_{32} 256 - \log_{16} 32 =$
- (a) $\log_2 8$ (b) 2 (c) $\log_{16} 224$ (d) $\log_{16} 8$ (e) 0.35 .

16. Which is closest to the area, in square miles, of the earth from the North Pole down to the Arctic Circle (23.5 degrees south of the North Pole)? Assume the earth is a sphere of radius 4000 miles.

- (a) 6×10^4 (b) 2×10^6 (c) 8×10^6 (d) 4×10^8 (e) 7×10^9

17. A dreidel is a 5 sided spinning die which is equally likely to land on each of its 5 sides (numbered 1, 2, 3, 4, 5). What is the probability that you will get a total of 7 if you spin a dreidel and roll a standard die and add the two results together?

- (a) $1/6$ (b) $1/5$ (c) $5/36$ (d) $7/25$ (e) $6/25$

18. Which number is closest to π^{3450} ?

- (a) 10^{1725} (b) 10^{1715} (c) 10^{345} (d) 2^{1725} (e) 2^{3450}

19. In which number base is the following computation correct:

$$18 * 17 = 291 ?$$

- (a) eleven (b) twelve (c) thirteen
(d) twenty-nine (e) forty-two

20. The three sides of a right triangle have integer lengths which form an arithmetic progression. One of the sides could be

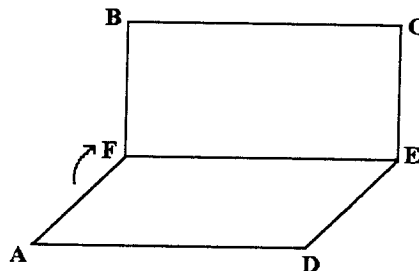
- (a) 17 (b) 22 (c) 58 (d) 77 (e) 95.

21. If $4^x - 4^{x-1} = 24$, find $8^x - 8^{x-1}$.

- (a) 36 (b) 48 (c) $24\sqrt{2}$ (d) $112\sqrt{2}$ (e) 576.

22. The square ABCD is folded in half. The perimeter of FBCE is 24 cm. Find the area of ABCD in square cm.

- (a) 8 (b) 32 (c) 40
(d) 64 (e) None of these



23. Before 1972, Britain had a number of unusual coins, including pennies, half-pennies (called ha'pennies) and three-penny pieces (called thrup'pennies). How many different ways could you give somebody 5 pence if you had three pennies, two thrup'pennies and six ha'pennies?

- (a) 2 (b) 3 (c) 5 (d) 6 (e) 7.

24. On their way back from a picnic, Susan, Franklin and Allison stop in to pick up some groceries. Susan buys 2 gallons of milk and a dozen eggs for \$5.72. Franklin buys 2 gallons of milk, 2 dozen eggs and a loaf of bread for exactly \$8.00. Allison buys a dozen eggs and a loaf of bread and gives the storekeeper \$3.00. How much change should she receive?

- (a) \$.14 (b) \$.50 (c) \$.72 (d) \$.87 (e) \$.88

25. A compartment measures exactly 12" wide by 24" deep by 5" high. How many cubes that are slightly less than 2 inches on a side can be stored in the compartment?

- (a) 144 (b) 180 (c) 360 (d) 720 (e) 1440

26. In the number base twenty, the following computation is correct:

$$\begin{array}{r} 1 \# \# \# \\ + \# 0 0 \\ \hline 2 0 \# \# \end{array}$$

What number does the digit # represent?

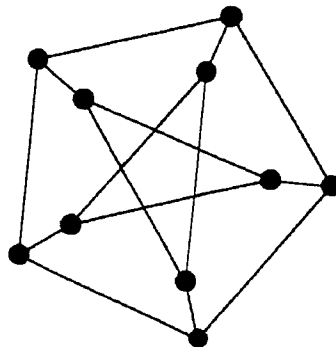
- (a) zero (b) one (c) six (d) ten (e) twelve.

27. In how many ways can a group of ten students be separated into two teams if the teams need not necessarily be of equal size?

- (a) 251 (b) 255 (c) 502 (d) 511 (e) 1023

28. How many different colors are needed to color each large dot on the graph below, so that no two dots which are connected by a straight line have the same color?

- (a) 1 (b) 2 (c) 3 (d) 4 (e) 5



29. A clock was correctly set at midnight January 1, just as 1996 began. It loses exactly 5 minutes each day of the year. What does the clock read one year later at midnight January 1, just as 1997 begins?

- (a) 5:00 (b) 5:20 (c) 5:30 (d) 5:35 (e) 5:45

30. A circle is inscribed in the figure formed by a quarter circle of radius one and the two radii. The radius of the inscribed circle is:

- (a) $\sqrt{2} - 1$ (b) $\frac{\sqrt{2}}{4}$ (c) $\frac{1}{2}$ (d) $\frac{1}{4}$ (e) $\sqrt{2} + 1$

31. The usual number of teeth in a human is 32. Some people, however, have lost some of their teeth. The average number of teeth in a group of people is 30.8. When four people having all of their teeth join the group, the average number of teeth rises to 31. How many people are **now** in the group?

- (a) 19 (b) 20 (c) 21 (d) 24 (e) 25

32. Two real numbers are selected at random from the interval $[0,10]$. Find the probability that the distance between the two numbers is > 5 .

- (a) $1/10$ (b) $1/5$ (c) $1/4$ (d) $1/3$ (e) $1/2$

33. A triangle has sides of length k , $k+5$ and $k+10$. One angle in the triangle has a cosine equal to -0.25 . Find k .

- (a) 5 (b) 10 (c) 15 (d) 20 (e) There is more than one such k .

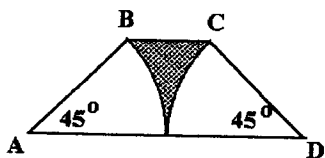
34. The line $2x - 3y = 5$ passes through how many points with both x and y positive integers less than 99?

- (a) 30 (b) 32 (c) 33 (d) 35 (e) 36

35. An acre contains 43560 square feet. A gallon of water measures 231 cubic inches. One foot equals 0.3048 meters. If one acre is flooded with two million gallons of water, the depth of the water in meters would be closest to:

- (a) 22.45 (b) $5/14$ (c) 6.1 (d) 2.72 (e) 1.87.

36. Two cities are 150 miles apart using a superhighway, and are 231 miles apart using a scenic route. The scenic route requires an average speed that is 18 mph slower than the average speed on the superhighway. The scenic route also takes three hours longer to travel. How long does the direct route take?
- (a) 2 hrs. 30 min. (b) 2 hrs. 40 min. (c) 2 hrs. 50 min.
(d) 3 hrs. exactly (e) 3 hrs. 6 min.
37. Twenty lots in a subdivision are to be sold. The first lot sells for \$30,000, and each succeeding lot sells for 98% of the previous lot's price. How much income is derived from the sale of these twenty lots?
- (a) \$478151 (b) \$498588 (c) \$500282 (d) \$524310 (e) \$588000
38. For how many different values of θ , with 0 degrees $< \theta < 360$ degrees is it true that: $2 \sin^2 3\theta = 1$?
- (a) 2 (b) 4 (c) 6 (d) 12 (e) 24
39. John catches a southbound #9 bus at 1:00 P.M. and notices that as he boards a northbound #9 bus is just arriving at the bus stop across the street. As he travels south, he sees 3 more northbound #9 buses and when he gets off downtown, he sees a 5th northbound #9 bus just leaving. The time he gets off the bus is 1:20 P.M. Assuming the buses leave downtown at regular intervals, how much time is there between northbound #9 buses, to the nearest minute?
- (a) 4 min. (b) 5 min. (c) 8 min. (d) 10 min. (e) 12 min.
40. Two congruent circular sectors are tangent at the point at which they meet side AD, as shown in the diagram. Each sector has radius one. The shaded area occupies about what percent of the the area of trapezoid ABCD?
- (a) 10% (b) 12% (c) 14% (d) 16% (e) 18%



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Karen H. Novotny
Grand Valley State University

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