

**FORTY FIRST ANNUAL
MICHIGAN MATHEMATICS PRIZE COMPETITION**

Sponsored by
The Michigan Section of the Mathematical Association of America

Part I

October 15, 1997

INSTRUCTIONS

(to be read aloud to the students by the supervisor or proctor)

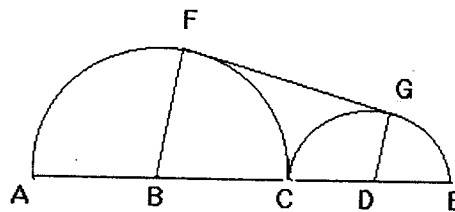
1. Your answer sheet will be graded by machine. Please read and follow carefully the instructions printed on the answer sheet. **Check to insure that your six-digit code number has been recorded correctly.** Do not make calculations on the answer sheet. Fill in circles completely and darkly.
2. Do as many problems as you can in the 100 minutes allowed. When the proctor requests you to stop, please quit working immediately and turn in your answer sheet.
3. Essentially all of the problems require some figuring. Do not be hasty in your judgments. For each problem you should work out ideas on scratch paper before selecting the answer.
4. You may be unfamiliar with some of the topics covered in this examination. You may skip over these and return to them later if you have time. Your score on the test will be the number correct. You are advised to guess an answer in those cases where you cannot determine an answer.
5. In each of the questions, five different possible responses are provided. In some cases the fifth alternative is listed "e) *none of these*". If you believe none of the first four alternatives to be correct, mark e) in such cases.
6. No one is permitted to explain to you the meaning of any question. Do not request anyone to break the rules of the competition. Scientific and graphing calculators are allowed on Part I this year, but the use of books, tables, slide rules, notes or any other aid is prohibited. If you have questions concerning the instructions, ask them now.
7. You may now open the test booklet and begin.

1. The length of the radius of a circle is increased by 10%. By what percent does the area increase?
a) 10% b) 14% c) 18% d) 21% e) depends on the original radius
2. The length of each side of a triangle is decreased by 30%. By what percent does the enclosed area decrease?
a) 9% b) 30% c) 49% d) 51% e) depends on the original side lengths
3. Recall that $n! = 1 \cdot 2 \cdot 3 \cdot 4 \cdots n$. Find the sum
 $(-1)^{1!} + (-1)^{2!} + (-1)^{3!} + \cdots + (-1)^{100!}$
a) 0 b) 100 c) 1 d) -1 e) 98
4. How many different chords can be drawn between 12 different points on the circumference of a circle?
a) 12 b) 132 c) 66 d) 12! e) 11!
5. A general equation of the family of lines perpendicular to $3y = 9x - 4$ is
a) $x + 3y = k$ b) $3y - x = k$ c) $x - 3y = k$ d) $3x + y = k$
e) $3x - y = k$
6. A tank is in the shape of an inverted right circular cone, with vertex at the bottom. Water flows into the tank until the depth of the water is half the height of the tank. What fraction of the tank's volume is still empty?
a) 15/16 b) 7/8 c) 1/4 d) 2/3 e) 1/2
7. The locus of points in the xy plane that are three units farther from the point $(-2,3)$ than they are from the point $(6,-4)$ is a
a) straight line b) parabola c) circle d) ellipse e) hyperbola
8. How would the base ten number 1997 be written in base three?
a) 2201222 b) 2202000 c) 221222 d) 2201022 e) none of these
9. If $f(x) = 9x + 1$ and $g(x) = x^2$, find all values of x which satisfy $f(g(x)) = g(f(x))$.
a) there are none b) 0 is the only value c) 0 and 0.25 d) 0.25 and -0.25
e) 0 and -0.25

10. If $\log_a 6 = 0.5$, then a is
- a) 36 b) 2^6 c) 2^{-6} d) 3 e) $\sqrt{6}$
11. The graph of $|x| + |y - 1| = 2$ is
- a) a circle b) an ellipse c) a triangle d) a square e) none of these
12. How many pairs of integers from this set are relatively prime (have no common factors) ?
- { 275, 546, 819, 875, 988 }
- a) three b) four c) five d) six e) seven
13. Six fair coins are tossed. What is the probability that three are heads and three are tails?
- a) $1/2$ b) $1/4$ c) $5/16$ d) $1/8$ e) $5/9$
14. How many real values of x satisfy the equation $\sqrt{2x-11} = 1 + \sqrt{x-2}$?
- a) none b) one c) two d) three e) infinitely many
15. The zeroes of the polynomial $2x^3 - 20x^2 + cx - 60$ are 2, 3, and 5. Find c .
- a) 72 b) 62 c) 48 d) -48 e) 12
16. Find the area of the trapezoid with bases of length two and eight, and with slant sides of length four and six.
- a) $(40/3)\sqrt{2}$ b) $(32/3)\sqrt{2}$ c) $14\sqrt{2}$ d) $40/3$ e) $10\sqrt{3}$
17. How many different angles A , with $0^\circ < A < 360^\circ$, satisfy $\sec A - 2 \cos A = 1$?
- a) none b) one c) two d) three e) more than three
18. A circular pizza with diameter 12 inches requires 14 ounces of dough. To make a circular pizza with the same thickness, but with diameter 16 inches, how many ounces of dough are required?
- a) about 19 oz. b) about 22 oz. c) about 25 oz. d) about 28 oz.
e) about 31 oz.

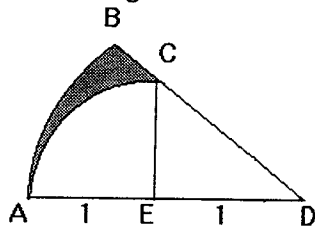
19. $\log_8 4 + \log_9 3 =$
- a) $\log_{17} 7$ b) $\log_{17} 12$ c) $5/6$ d) 1 e) $7/6$
20. Three nonzero numbers a , b , and c satisfy $b - a = c - b$, $b/(a + 1) = c/b$, and $b/a = (c + 2)/b$. Then b must have the value:
- a) 3 b) 8 c) 12 d) -6 e) some other number
21. A wheel has radius 15 inches. About how many revolutions would the wheel make on a four mile trip?
- a) 1793 b) 2689 c) 2964 d) 3586 e) 5378
22. A bee always gathers pollen from its favorite flower garden. It leaves the hive and flies directly to the garden at the rate of five miles per hour. It returns to the hive at the rate of four miles per hour. If its total flight time was 90 minutes, how far from the hive is the flower garden?
- a) $2\frac{2}{3}$ miles b) 3 miles c) $3\frac{1}{3}$ miles d) 4 miles
e) cannot be determined
23. If $0 < a < c$ and $a - b + c = 0$, find the larger root of the quadratic equation $ax^2 + bx + c = 0$.
- a) -1 b) 1 c) 0 d) c/a e) cannot be determined
24. Solve for y in terms of x if $\log_{10} x = 6 + 3 \log_{10} y$.
- a) $x - 6$ b) $(x - 6)/10^{1/3}$ c) $x^{1/3}/100$ d) $x/3 - 200$ e) none of these
25. If $k > 0$, which statement best describes those m for which the graphs of $y = mx - k$ and $y = kx^2$ will not intersect?
- a) $-2k < m < 2k$ b) $-k < m < 3k$ c) $-3k < m < k$ d) $2k < m < 3k$
e) $k < m < 3k$
26. Two dice are rolled. What is the probability that the numbers on the two dice are consecutive integers?
- a) $1/6$ b) $1/4$ c) $1/3$ d) $4/9$ e) $5/18$

27. A regular heptagon (seven-sided polygon) is inscribed in a circle. The area of the heptagon is about what percent of the area of the circle?
- a) 81% b) 84% c) 87% d) 90% e) 93%
28. A commuter train 200 meters long speeds along at the rate of 200 km/h. It passes through a 200 meter long tunnel. How much time does it take the train to pass completely through the tunnel?
- a) 3.6 seconds b) 7.2 seconds c) 10.8 seconds d) 36 seconds
e) 72 seconds
29. The inequality $x + |3x| < 10$ is equivalent to:
- a) $x < 2.5$ b) $|x| < 5$ c) $|x + 5| < 2.5$ d) $|x + 1.25| < 3.75$
e) none of these
30. Al would take six hours to paint a wall working alone, while Bob would take four hours to paint it and Chuck would take three hours to paint it. Al starts painting at noon. At 1:00, Bob joins him. Chuck joins them at 1:30. When do they finish painting the wall?
- a) 2:15 b) 2:20 c) 2:30 d) 2:40 e) 2:45
31. Four years ago, Jessica was twice as old as Heather. Seven years from now, the sum of their ages will be 79. How much older than Heather is Jessica?
- a) 10 years b) 14 years c) 16 years d) 19 years e) none of these
32. The semicircle centered at B has radius three, while the semicircle centered at D has radius one. ABCDE is a straight line and line FG is tangent to both semicircles. Find the area of quadrilateral BFGD.



- a) $3\sqrt{5}$ b) $4\sqrt{3}$ c) 7 d) $5\sqrt{2}$ e) $\sqrt{51}$

33. The distance along the surface of the earth from Detroit to Miami is 1318 miles. If the earth is a sphere with radius 3975 miles, what angle is formed at the center of the earth when lines are drawn to the center from both Detroit and Miami?
- a) 10° b) 17° c) 19° d) 22° e) 24°
34. If $\text{Arctan } M + \text{Arctan } N = \pi/3$, then $M + N$ equals
- a) $\sqrt{3}$ b) $MN\sqrt{3}$ c) $\sqrt{3MN}$ d) $\sqrt{3} / MN$ e) $\sqrt{3} (1 - MN)$
35. Find $\sum_{k=1}^n k^2$ divided by $\sum_{k=1}^n k$
- a) n^2 b) $(n^2 + n)/2$ c) $(2n + 1)/3$ d) $(n^2 + 2n)/3$
e) the value depends on k
36. If $\log_{\tan A} (\sec A) = 2$, find $\cos A$.
- a) $1/2$ b) $\sqrt{5}/3$ c) $\sqrt{3}/2$ d) $(1 + \sqrt{5})/2$ e) $2/(1 + \sqrt{5})$
37. Either Jim plays tennis and Kevin doesn't play tennis, or else Kevin plays tennis and Ed doesn't play tennis. Mary plays golf unless Jim doesn't play tennis. Mary doesn't play golf. So:
- a) Ed doesn't play tennis.
b) Kevin doesn't play tennis.
c) Jim plays tennis.
d) Ed plays tennis.
e) none of these conclusions are logically valid.
38. In the diagram below, AB is a circular arc centered at D, and AC is a circular arc centered at E. Point E is the midpoint of line segment AD, and CE is perpendicular to AD. Both AE and ED have length one unit. Find the shaded area.

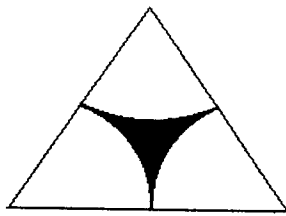


- a) $(5\pi - 6)/12$ b) $(\pi - 1)/8$ c) $(2\pi - 5)/6$ d) $(2\pi - 6)$ e) $(\pi - 2)/4$

39. What is x if we know that $x = \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{\dots}}}}$?

- a) $\sqrt{3}$ b) 2 c) $\sqrt{5}$ d) 3 e) $1 + \sqrt{2}$

40. An equilateral triangle has three congruent circular arcs inside, each centered at a vertex of the triangle. The shaded area is about what percent of the area of the triangle?



- a) 7% b) 9% c) 11% d) 13% e) 15%

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Mathematical Association of America

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ACKNOWLEDGMENTS

The following individuals, corporations and professional organizations have contributed
generously to this competition:

Addison-Wesley Publishing Co.
Ford Motor Company
Jerome J. Kohel
John Wiley & Sons
Kuhlman Corporation
Grand Valley State University
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Matilda R. Wilson Fund
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