1. A sequence a_n is defined by $a_1 = 1$ and $a_n = 2a_{n-1} - 1$ for $n \geq 2$. What is a_{2009}?

   A: 1 B: 2009 C: 2^{2009} D: $2^{2009} - 1$ E: none of the others

2. A polynomial $P(x)$ is known to be of the form

$$P(x) = x^{19} - 7x^{18} + \ldots - 13,$$

   where the ellipsis $(\ldots)$ represents unknown intermediate terms. It is also known that all roots of $P(x)$ are integers. The largest root of $P(x)$ is

   A: 7 B: 13 C: 6 D: not deducible from the given information. E: none of the others

3. For every two real numbers, a and b, the operation $<>$ is defined by $a <> b = \frac{b-a}{a+b}$. The value of $(3 <> 9) <> 8$ is

   A: $\frac{15}{17}$ B: $\frac{9}{5}$ C: $\frac{12}{19}$ D: $\frac{1}{2}$ E: none of the others

4. How many solutions are there to $\log_{10} x + \log_{10}(x + 9) = 1$?

   A: 0 B: 1 C: 2 D: infinitely many E: none of the others

5. Ada, Eddie, Jennifer were the only competitors in a 5K race where each person finished in a different position. The three people, all notorious liars, reported the results of the race as follows.

   Ada: I finished second and Eddie finished last.
   Eddie: I finished second or Jennifer finished last.
   Jennifer: I finished second but Ada finished last.

   All three of the above statements are false. What was the order of finish in the race?

   A: Ada, Jennifer, Eddie.
   B: Eddie, Ada, Jennifer.
   C: Jennifer, Eddie, Ada.
   D: There is not enough information to determine the order of finish.
   E: None of the others.

6. For every pair of non-zero real numbers a and b, form the sum

$$1 + \frac{a}{|a|} + \frac{b}{|b|} + \frac{ab}{|ab|}.$$

The set of all numbers created by this process is

A: $\{0, 2, 4\}$ B: $\{4\}$ C: $\{0, 4\}$ D: $\{0\}$ E: none of the others

7. Let$f(x)$ be an odd function defined on $(-\infty, \infty)$ and $g(x)$ be an even function defined on $(-\infty, \infty)$. If

$$f(x) - g(x) = x^2 + 5x + 7,$$

then $f(x) + g(x)$ is equal to

A: $-x^2 - 5x + 7$ B: $x^2 + 5x - 7$ C: $-x^2 + 5x - 7$ D: $x^2 - 5x + 7$
E: none of the others

8. Suppose $\sin\alpha + \cos\alpha = 1/5$. What is $\sin^3\alpha + \cos^3\alpha$?

A: $\frac{37}{125}$ B: $\frac{1}{125}$ C: $\frac{91}{125}$ D: $\frac{47}{125}$ E: none of the others

9. In the picture below, the triangle ABC is an isosceles triangle with a right angle at C. The triangles ADB and CDB both have area 1 in^2. The segment DE is perpendicular to AB. What is the length (in inches) of DE?

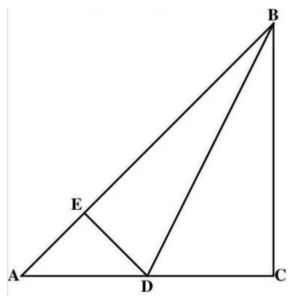

Figure 1: Figure for Question 9

A: $\frac{1}{\sqrt{3}}$ B: $\frac{2}{\sqrt{2}}$ C: $\frac{1}{\sqrt{2}}$ D: $\frac{2}{\sqrt{3}}$ E: none of the others

10. There are five numbers. Their average is 8, the median is 10 and the number that appears most often is 2. What is the sum of the two largest numbers?

A: 25 B: 26 C: 24 D: 23 E: none of the others

11. What is the angle between the hour-hand and minute-hand of a clock at 3:15 in degrees?

    A: 8 B: 7.5 C: 7.75 D: 7.25 E: none of the others

12. A digital score board with 6 cells can display any scores from 0 to $999,999$ but it must have a number for each cell. So 0 will be represented as 000000. How many times does the number 1 appear in the $1,000,000$ representations? For example, 1 appears 4 times in 001111.

    A: $99,999$ B: $199,999$ C: $200,000$ D: $600,000$ E: none of the others

13. Two coins are placed randomly, at different locations, in a 5x6 grid. What is the probability that the two coins are not in the same row or column?

    A: $\frac{3}{5}$ B: $\frac{2}{3}$ C: $\frac{20}{29}$ D: $\frac{7}{10}$ E: none of the others

14. Given that a and b are positive integers that satisfy $1 \le a, b \le 6$. Find the probability that both roots of the following equation are positive:

$$x^2 - 2(a-3)x - b^2 + 9 = 0.$$

    A: $\frac{1}{9}$ B: $\frac{1}{18}$ C: $\frac{1}{6}$ D: $\frac{13}{18}$ E: none of the others

15. Let

$$f(x) = 1 - \log_x 2 + \log_{x^2} 9 - \log_{x^3} 64.$$

    Find the range of x that makes $f(x) < 0$.

    A: $\left(1, \frac{8}{3}\right)$ B: $(0,1)$ C: $(1,\infty)$ D: $\left(\frac{8}{3}, \infty\right)$ E: none of the others

16. Rhombus $ABCD$ has side length to be a. Point O is a point on the diagonal AC with $\|OA\| = a$, and $\|OB\| = \|OC\| = \|OD\| = 1$, where $\|\cdot\|$ represents a length. Find a.

    A: 2 B: $\frac{\sqrt{5}-1}{2}$ C: 1 D: $\frac{\sqrt{5}+1}{2}$ E: none of the others

17. Let set $A = [-2, 4)$ and set $B = \{x \ : \ x^2 - ax - 4 \le 0\}$. If $B \subseteq A$, then the range of a is:

    A: $[-1,2]$ B: $[0,3)$ C: $[-1,2)$ D: $[0,3]$ E: none of the others

18. Let $f(x) = 11 - 8\sin x - 2\cos^2 x$, where x is a real number. Denote the maximum and minimum values of $f(x)$ by a and b, respectively. Find $\frac{a-1}{b}$.

    A: 6 B: 18 C: 5 D: 0 E: none of the others

19. Let a and b be non-zero real numbers which satisfy

$$|2a-4|+|b+2|+\sqrt{(a-3)b^2}+4=2a.$$

Find $a+b$.

A: 22 B: 0 C: -1 D: 1 E: none of the others

20. Let n be a positive integer. The function $\sigma(n)$ is the sum of the positive divisors of n. For example, $\sigma(4)=7, \sigma(5)=6$, and $\sigma(6)=12$. The number of odd integers between 1 and 100, inclusive, for which $\sigma(n)$ is odd is

A: 10 B: 16 C: 5 D: 9 E: none of the others

21. Professor Lewis will be on sabbatical for 255 days. He has asked Professor Cheng to water his flowers while he is gone. Professor Cheng must water the rhododendrons every 3^{rd} day, the petunias every 5^{th} day, and the begonias every $17^{\text{th}}day$. How many days will Professor Cheng not have to water any flowers?

A: 103 B: 104 C: 129 D: 128 E: none of the others

22. Five girls–Alice, Barbara, Carolyn, Deena, and Eva–sang songs in concert as trios, with two girls sitting out each time. Alice sang 10 songs, which was more than any other girl. Barbara sang 9 songs, which was more than any girl except Alice. Eva sang 6 songs, which was fewer than any other girl. How many songs were sung by at least one of the trios?

A: 13 B: 14 C: 16 D: Insufficient information to get the answer. E: none of the others

23. The line joining the midpoints of the diagonals of a trapezoid has length 5. If the longer base has length 100, then the shorter base has length

A: 95. B: 90. C: 100. D: 85. E: none of the others

24. Suppose f is a function that satisfies $f(x)+3f(10-x)=2x$ for all real number x. Find $f\left(\frac{15}{2}\right)$.

A: 1 B: -1 C: 0 D: $\frac{15}{2}$ E: none of the others

25. An equilateral triangle and a square have the same perimeter. What is the ratio of the area of the triangle to the area of the square?

A: $4\sqrt{3}/9$ B: $\sqrt{3}/3$ C: $2\sqrt{3}/9$ D: $5\sqrt{3}/9$ E: none of the others

26. Find the number of positive divisors of 7!.

A: 55 B: 60 C: 80 D: 49 E: none of the others

27. Evaluate

$$\frac{1}{2+\frac{1}{2+\frac{1}{2+\frac{1}{2+\cdots}}}}.$$

A: $\sqrt{2}$ B: $\sqrt{2}+1$ C: $\sqrt{2}-1$ D: $2\sqrt{2}$ E: none of the others

28. Find $S = \sum_{n=1}^{2009} \frac{5^{2010}}{25^n + 5^{2010}}$.

A: $\frac{2009}{2}$ B: $\frac{1}{5}$ C: $\frac{2009}{5}$ D: 1 E: none of the others

29. What is the highest power of 35 that divides 343!?

A: 57 B: 70 C: 83 D: 43 E: none of the others

30. The midpoints of the sides of a triangle in the xy-plane are (1, 1), (4, 3), and (3, 5). Find the area of the triangle in units square.

A: 14 B: 18 C: 22 D: 16 E: none of the others

31. Find the area of the region whose points satisfy the inequality $m \leq |x| + |y| \leq 2m$ where m is a positive number.

A: $3m^2$ B: $6m^2$ C: $8m^2$ D: $6m^2 + 6m$ E: none of the others

32. Find the number of 3-digit numbers where none of the digits is 0 and the sum of the digits is 10.

A: 34 B: 32 C: 36 D: 38 E: none of the others

33. A triangle has vertices A, B and C, and the respective opposite sides have lengths a, b and c. This triangle is inscribed in a circle of radius R. If $b = c = 1$ and the altitude from A to side BC has length $\sqrt{\frac{3}{2}}$, then R equals:

A: $\frac{1}{\sqrt{3}}$ B: $\frac{2}{\sqrt{3}}$ C: $\frac{\sqrt{3}}{2}$ D: 1 E: none of the others

34. A square has vertices $(0,0),(0,4),(4,0)$ and $(4,4)$. Find the equation of the line that satisfy the following conditions:

- it passes through the point $(0,1)$
- the area above the line and inside the square is $\frac{1}{3}$ the area of the area below the line and inside the square.

A: $7y+8x=7$ B: $7y-8x=8$ C: $8y-9x=8$ D: $8y-7x=8$
E: none of the others

35. Given $f(x)=12x^2+8x-9$ and $g(x)=-9x^2+14x+29$, denote the sums of the roots of $f(x)$ and $g(x)$ by a and b, respectively. The sum of the roots of $h(x)=ax^2+bx+ab$ is?

A: $\frac{3}{4}$ B: $\frac{4}{7}$ C: $\frac{16}{21}$ D: $\frac{7}{3}$ E: none of the others

36. Lisa wants to compose a 7-digit number whose product of the digits is 25000. How many such numbers can she make?

A: 30 B: 84 C: 1 D: 0 E: none of the others

37. The integer a leaves a remainder of 4 when divided by 5. The integer b leaves a remainder of 2 when divided by 5. What is the remainder of a^5+b^5 when divided by 5?

A: 2 B: 1 C: 3 D: 4 E: none of the others

38. Find the number of solutions (x,y) to the equation $x^2+xy+2y^2=29$ where both x and y are integers.

A: 0 B: 2 C: 4 D: 3 E: none of the others

39. The range of m such that the curves $y=\sqrt{-x^2-2x}$ and $x+y-m=0$ have two intersection points is

A: $(0,\sqrt{2}-1)$ B: $[0,\sqrt{2}-1)$ C: $(-\sqrt{2}-1,\sqrt{2})$ D: $(-2,\sqrt{2}-1)$
E: none of the others

40. Find the area of the region in the plane where each point (x,y) satisfies the equation:

$$(x-4\cos\theta)^2+(y-4\sin\theta)^2=4, \quad 0\le\theta<2\pi.$$

A: 36π B: 16π C: 32π D: 20π E: none of the others